PH102 Tutorial Sheet 3 (Jan 16, 2015)
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1. For a given vector $\vec{\boldsymbol{A}} = x^2\hat{\boldsymbol{\imath}} + y^2\hat{\boldsymbol{\jmath}} + z^2\hat{\boldsymbol{k}}$ verify the divergence theorem over the cube of side length unity having four of its vertices at (0,0,0), 0,0,1), (01,0) and (1,0,0).
2. Verify the divergence theorem over the volume of an ice-cream cone as shown in fig1 for the vector field $\vec{\boldsymbol{v}} = r^2 sin\theta\hat{\boldsymbol{r}} + 4r^2 cos\theta\hat{\boldsymbol{\theta}} + r^2 tan\theta\hat{\boldsymbol{\varphi}}$.
3. A closed cylindrical shell is defined as ρ=a and ρ=2a and z=±aπ/2. Calculate the total flux coming out of this closed surface for a vector field given by
   $\vec{F} = \frac{\rho}{a}\cos(\lambda z)\hat{\varrho} + \sin(\lambda z)\hat{k}$
4. Verify the stokes theorem for the vector field $\vec{\boldsymbol{A}} = y\hat{\boldsymbol{\imath}} - x\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}=$ over the hemispherical surface $x^2 + y^2 + z^2 = a^2$ *and* $z \geq 0$.(flat surface is open)
5. Show that vector field $\vec{\boldsymbol{B}} = (z^2 + 2xy)\hat{\boldsymbol{\imath}} + (x^2 + 2yz)\hat{\boldsymbol{\jmath}} + (y^2 + 2zx)\hat{\boldsymbol{k}}$ is conservative. Calculate it's line integration along any line joining the points (1,1,1) and (1,2,2).
6. A surface is formed by the cylinder $x^2 + y^2 = a^2, 0 \leq z \leq h$. The top surface of the cylinder is closed and the bottom surface is open. Verify the stokes theorem for the vector $\vec{F} = -y\hat{\imath} + x\hat{\jmath} + x^2 k$
7. Verify the stokes theorem over the surface as shown in Fig 2 for the vector field
   $\vec{\boldsymbol{V}} = rcos^2\theta\hat{\boldsymbol{r}} - rcos\theta sin\theta\hat{\boldsymbol{\theta}} + 3r\hat{\boldsymbol{\varphi}}$ (recall the problem 7 of tut sheet 1)
8. Evaluate the following integrals:
   i. $\int_{-\infty}^{+\infty} \ln(x+3)\,\delta(x+2)dx$
   ii. $\int_{-\infty}^{a} \delta(x-b)dx$
   iii. $\int_{0}^{2} (x^2 + 3x + 2)\delta(1-x)dx$
9. i. show that $x\frac{d}{dx}(\delta(x)) = -\delta(x)$

   ii. Let f(x) be a step function defined as $f(x) = \begin{cases} 1, for\ x > 0 \\ 0,\ for\ x \leq 0 \end{cases}$, work out $\frac{df}{dx}$. Plot the function *f(x)* as well its derivative as a function of x.

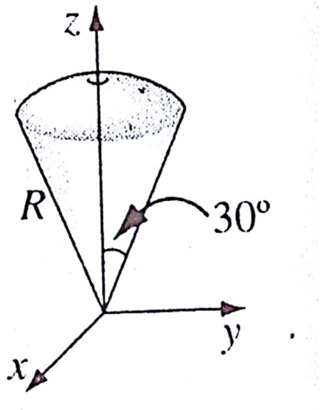

Fig 1

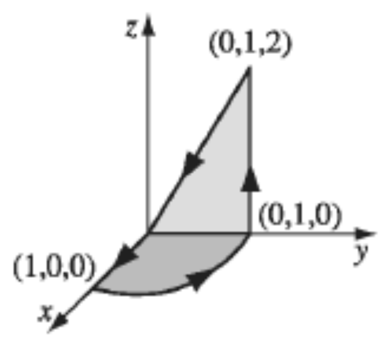

Fig 2